Measuring and managing market risk

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Investment management is largely concerned with risk management. In the management of the Petroleum Fund, considerable emphasis is therefore placed on measuring and managing risk. However, there are no exact measurement methods. This article provides an account of the most important methods and models used in the Petroleum Fund. Since all models contain sources of error, Norges Bank uses a broad-based set of tools in its regular analyses of the risk associated with management.

Absolute and relative risk

An investor cannot know in advance what return he will achieve on his investments in financial instruments. His market risk is the uncertainty as to how the market values of the instruments will actually develop. The investor may choose to invest in portfolios with a greater or lesser degree of market risk. One standard hypothesis in financial theory is that the expected return exceeds the risk-free interest rate only if there is risk associated with the investments.

There is an important difference between absolute and relative risk, or volatility - a word that is often used synonymously with risk in this context. Absolute risk (volatility) indicates the amplitude of the variations in portfolio value that can normally be expected. Relative risk, or tracking error, expresses the amplitude of the variations in portfolio value that can normally be expected relative to another portfolio (the benchmark portfolio).

If absolute risk is measured as 10 per cent for a portfolio with a value of NOK 100 billion, this means that after one year there is a high probability that the value of the portfolio will lie in the interval NOK 90-110 billion. Normal variations in the securities market can accordingly result in anything from a loss of NOK 10 billion to a gain of NOK 10 billion.

The Ministry of Finance has defined a benchmark portfolio which specifies how the capital in the Petroleum Fund shall normally be invested. This benchmark consists of equities and fixed income instruments in specific proportions issued in a number of countries. The Ministry of Finance has drawn up a set of rules that clearly define how this portfolio is to be composed. A central rule is that equities shall constitute 40 per cent, while fixed income instruments shall constitute 60 per cent. This distribution between asset classes is the most important rule limiting the absolute risk to which the Fund is exposed. In 2002, the absolute risk in the Petroleum Fund’s benchmark portfolio was 8-9 per cent.

The Ministry has placed limits on the amount of the risk Norges Bank may take relative to the benchmark portfolio. The Ministry has thus set limits for tracking error, whereas absolute volatility is limited indirectly by the choice of benchmark. The tracking error limits ensure that the return on the Petroleum Fund does not deviate too much from the return on the benchmark portfolio.

The tracking error limit is set at 1.5 percentage points. If the Bank fully utilises this limit, it means that the value of the Fund one year ahead will normally not differ by more than 1.5 per cent from the value of the benchmark. At the end of December 2002, the expected tracking error for the Fund was measured as 0.4 percentage point. It follows that the value of the Fund in one year’s time will with a high degree of probability not deviate more than 0.4 per cent from the value of the benchmark.

This illustrates that the bulk of the absolute market risk in the Petroleum Fund depends on the benchmark. It also means that the performance of the Fund is largely determined by the composition of the benchmark. Norges Bank can only exert a limited influence on the overall market risk and performance of the Fund.

Standard deviation as a measure of risk

The most common measure of risk is the standard deviation of the return. The absolute volatility and relative volatility (tracking error) of the Petroleum Fund are measured by estimating the standard deviation of the absolute return on the Fund and the standard deviation of the difference between the return on the Fund and the return on the benchmark.

The standard deviation provides some indication of the variation in returns that may be expected during normal periods. Negative rates of return count just as much as positive rates of return, and the standard deviation is therefore a symmetrical measure of risk. There are financial instruments that do not have a symmetrical return profile, including options and bonds with credit risk. For this type of instrument, the standard deviation may be a very inaccurate measure of risk. Norges Bank invests in a very limited extent in options, but has a large holding of fixed income instruments with credit risk.

The Petroleum Fund’s portfolio contains equities and fixed income instruments issued in many different countries and by a substantial number of issuers. In order to be able to estimate the risk associated with the total portfolio, it must be possible to estimate the standard deviation of the returns on all securities in the portfolio. In addition, it must be possible to estimate the covariation of the returns on the various securities. When the return on share A is higher than the average for this share, we must know whether the return on bond B can also be expected to be higher than average for this bond, or whether it can be expected to be lower. Correlation coefficients, which are a measure of covariation, are figures in the interval from -1 to +1. A positive correlation coefficient means that on average a high return on one security is accompanied by a high return on another security. When the correlation coefficient is negative, a high return on one security is accompanied by a low return on the other security.
If there is perfect correlation between the returns on two securities (correlation coefficient equals 1), then the standard deviation (risk) of a portfolio consisting of the two securities is equal to the sum of the standard deviations of the individual securities. If the returns are not perfectly correlated, the standard deviation of the portfolio will be lower than the sum of the standard deviations. This result applies generally. When a portfolio consists of a number of equities and fixed income instruments, the risk associated with the portfolio will typically be substantially lower than the sum of the risk of the individual securities in the portfolio. This is because when the return on some securities is negative, it is often positive for others. As a rule, broadly diversified portfolios have much lower risk than portfolios consisting of only a few securities. However, not all risk is eliminated. Even a broad-based equity portfolio is exposed to general rises and falls in the equity market (systematic risk). This risk can only be reduced by investing less in equities and more in less risky assets such as bonds or bank deposits.

**Methods for estimating risk**

The standard deviation of the return relative to the benchmark (tracking error) is an important measure of risk for Norges Bank. Various methods have been developed for calculating standard deviation. The most common is the parametric method. Methods such as “Monte Carlo” and historical simulation are also widely used.

**Parametric method**

With the parametric method, the standard deviation of the portfolios is measured using the standard deviations of and correlations between the various securities in the actual and benchmark portfolios. As a rule, the standard deviations of the individual securities are calculated on the basis of historical performance. These historical rates of return also form the basis for calculating the covariation of the different securities.

The absolute risk of the fund is calculated by multiplying the standard deviations and correlations by the market values of the holdings of the various securities. Relative risk is calculated by multiplying the standard deviations and correlations by the values of the differential holdings. The Fund’s differential holding in a limited company is the difference between the number of shares the Fund owns in this company and the number of shares in the benchmark.

When standard deviations and correlations have been estimated and holding sizes determined, absolute volatility and tracking error can easily be calculated. However, the parametric computation method has some weaknesses. One example is the manner in which options are treated.

The parametric method presupposes that all financial instruments have a return profile that is linear or symmetrical. An equity is an example of a linear instrument. If we have 200 shares, and the share price rises by NOK 10, we earn NOK 2000 (200 multiplied by NOK 10). Similarly, if the price falls by NOK 10, we will lose NOK 2000 on this position. An option to purchase the share, which gives the buyer a right, but not an obligation, to purchase at a specific price, does not have a linear return profile. If the share price rises, the value of the option increases as though the potential buyer owned the share. If the share price falls, the value of the option also falls. But the fall in the value of the option is limited, in that the value of the option cannot fall below zero. Similar skewnesses apply generally to all types of option.

In the parametric method, option positions are converted to positions in the underlying asset using a constant ratio. Options are thus assumed to have a linear return profile, even though this is not correct.

**Monte Carlo simulations**

To compensate for this weakness, among others, alternative methods have been developed for calculating the standard deviation. One such method is based on the Monte Carlo (MC) simulation technique. Assume that both the benchmark portfolio and the actual portfolio consist of 100 securities, but with different holdings. The MC method consists in drawing 100 random rates of return from a specified probability distribution (often the normal distribution). Then new prices are computed for the 100 securities, and the value of the two portfolios is computed again. Then a further 100 random rates of return are drawn, new prices are computed on the basis of current prices, and new portfolio values are computed. A large number of these draws are made, and from the set of different valuation changes, absolute volatility and tracking error can be calculated.

The draws are made in such a way that the historical covariation between the returns on the various securities is retained, and also such that the difference in the rates of return of the various securities reflects historical differences in risk. One advantage of this technique is that for each new price that is simulated, for example for a share, an option on this share can be valued on the basis of the new price and an option pricing method. This means that options can be valued exactly, irrespective of the size of the price change simulated for the underlying share.

The MC simulation method may require a longer computational time than the parametric method, but is more accurate if the portfolio contains options.

Chart 1 presents the results of the two methods for two real portfolios that both contain options, and for the sum of the two portfolios. The standard deviations are given in millions of NOK, and represent the intervals within which the return will fall with a probability of about 68 per cent. The chart also shows how large the intervals must be for there to be a 95 per cent probability that the result will fall within the intervals.
Chart 1: Estimated risk with the parametric and Monte Carlo methods. In millions of NOK

For the portfolios in Chart 1, there is not very much difference between the parametric method and the MC simulation. This is because the portfolios only contain small option positions. The further out one comes in the probability distribution tail, the greater the difference; it is larger for a 95 per cent confidence interval than for one standard deviation.

Historical simulations
The historical simulation method is based directly on actual rates of return. Assume that we have five years of monthly rates of return for all securities in the actual and benchmark portfolios. The starting point is the market values of the two portfolios with current prices for the securities they contain. New (simulated) prices are computed using the first historical month of return rates for each security. This provides a basis for calculating new market values for the portfolios, and a rate of return for each portfolio. Similarly, rates of return are computed for the portfolios using historical rates of return for each security from the second month. This method is repeated for each set of historical rates of return for the securities. Finally, an absolute volatility can be calculated for each portfolio, and a tracking error (standard deviation) for the actual portfolio relative to the benchmark portfolio.

The advantage of historical simulation is that it does not link a measure of risk to a mathematically defined probability distribution. The disadvantage is that the future return may differ considerably from the rates of return in the period upon which the simulation is based.

As with MC simulations, an exact price is computed for financial derivatives, given the change in price of the underlying asset. Chart 2 shows the risk for two portfolios and for the sum of the portfolios, computed using the parametric method and historical simulation, respectively. Figures are in millions of NOK and are based on annual rates of return. There are substantial differences. The calculations illustrate the importance of the assumptions concerning probability distribution for the result, and stress that there is no final answer.

Chart 2: Estimated risk using the parametric method and historical simulation In millions of NOK

Complementary methods
The three models presented above complement one another. Standard risk measurements in the Petroleum Fund are based on the parametric method. It is fast and functions well as long as the portfolio contains a limited volume of instruments with built-in optionality, as is currently the case for the Petroleum Fund. So far, options have not been used much in management. The Monte Carlo simulation method comes into its own when the portfolio contains options. A few sub-portfolios in the Petroleum Fund contain options, and for these portfolios it is useful to supplement the parametric risk calculations with MC calculations. When Norges Bank calculates risk using the parametric method and Monte Carlo simulations, the return is assumed to be normally distributed. In some periods, the return on financial assets varies more than can be expected on the basis of normally distributed rates of return. Historical simulations use the historical return distribution. This can capture both extreme price movements which are not captured by the normal distribution, and any asymmetries in the rates of return.
Choice of parameters

Norges Bank Investment Management changed its risk measurement system in September 2002. The supplier is now RiskMetrics, and the system is called RiskManager. One important reason for changing systems was the expansion of the benchmark portfolio to include mortgage-backed securities. The new system has generally good coverage of the various fixed income instruments, and is also relatively simple to operate.

Suppliers of risk models build various degrees of flexibility into their risk models. One factor that is important in connection with the computation of both absolute volatility and tracking error is the weighting of various historical observations. We can choose to attach less weight to old observations than to new ones. The idea is that more recent rates of return contain more information about developments in the near future than old ones. Norges Bank’s computations using RiskManager are based on this approach. In practice this means that fewer observations are used to calculate the standard deviation than would be used if old observations were to be given the same weight as new ones.

The implication for the calculation of absolute volatility is unambiguous: in periods when market volatility in the very recent past has been historically high, the weighting down of old observations will result in higher calculated risk than equal weighting would do. When volatility is historically low, down-weighting of old observations will result in lower calculated risk than equal weighting would.

However, the implication for tracking error is not clearcut. It is not necessarily the case that in a period of historically high market volatility, a higher tracking error will be obtained by down-weighting of old observations than by equal weighting of old and recent observations. The deciding factor is how the actual portfolio and the benchmark would have co-varied during the period.

There are differences between daily, weekly and monthly rates of return. For example, volatility and correlation calculated from daily rates of return may differ from the results yielded by monthly rates of return. Daily rates of return will often yield higher volatilities than monthly rates of return. Norges Bank uses daily rates of return. The most important reason for this is that the possibility of using monthly rates of return is relatively new in RiskManager.

Stress testing

Stress tests are used to supplement standard risk computations. One problem with ordinary risk calculations is that they only measure the size of normal variations, and thus do not reveal the effect of extreme events. It is difficult to estimate the probability of extreme events, precisely because there are so few observations of them. This also makes it difficult to use simulation techniques, precisely because a large number of draws are required in the interests of accuracy. Extreme events are therefore often analysed by means of stress testing.

Stress tests can be linked to major crises, like wars, political instability, natural disasters and speculative attacks on various currencies. The tests can also be linked to periods with speculative bubbles in securities markets, or to changes in, for example, monetary policy management.

Stress tests are normally performed in two stages. First, the stress event is chosen. There is no established standard for such choices. In our risk system, we can choose between historical scenarios such as the Asian crisis (1997), Black Monday (1987), the Gulf War (1990), the peso crisis (1995), the Russian devaluation (1998), the World Trade Center (2001) and the crisis in Brazil (also called the Bovespa crisis), 1999. New historical scenarios are added at regular intervals. In addition we can define our own stress events.

The portfolio is then valued under the stress scenario, taking the present value of the portfolio as the starting point. It is then revalued on the basis of developments in the equity, fixed income and foreign exchange markets during the historical stress period. The difference between the present value of the portfolio and a new value, given that markets developed as in the historical scenario, is a measure of the loss (gain) that would be achieved if a similar situation arose. However, there is no indication of how probable such a scenario is.

Stress tests can indicate how large the hypothetical loss (gain) would be in NOK, and how large it would be compared with the loss (gain) in the benchmark portfolio.

Table 1 shows the impact on the value of the Petroleum Fund of two extreme events. The first is associated with the crisis in Brazil in 1999, which had a contagion effect on the US equity market. The second event is the terrorist attack on the World Trade Center in 2001. The first line shows how much the value of the Petroleum Fund would have fallen, while the second shows the fall in value of the benchmark. The third line shows the difference between the two portfolios, which is positive in this case. The table shows that the Petroleum Fund would not have done quite as badly as the benchmark in these scenarios.

Table 1: Stress tests for the Petroleum Fund in historical scenarios.

<table>
<thead>
<tr>
<th>Historical scenarios</th>
<th>Bovespa down 25%</th>
<th>World Trade Center</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Petroleum Fund</td>
<td>-12 178</td>
<td>-6 758</td>
</tr>
<tr>
<td>Benchmark portfolio</td>
<td>-12 959</td>
<td>-6 846</td>
</tr>
<tr>
<td>Relative result</td>
<td>781</td>
<td>88</td>
</tr>
</tbody>
</table>
Validation of models

It is difficult to have a clear idea in advance of how good the estimates of absolute volatility and tracking error are. We cannot expect very accurate estimates of actual volatility in any one period, but over time the average of the estimates should indicate a risk that is approximately equal to actual risk. Investigating whether this is the case is called validating the model.

Validation of risk models presupposes access to a large number of observations of predicted risk and actual variations. Charts 3-5 show how BARRA, Norges Bank’s previous risk system, predicted relative risk at the beginning of each month, and what the excess return actually turned out to be that month. Chart 3 shows the 90 per cent confidence interval, based on risk predictions and actual excess return in the individual period for the total portfolio. The next two charts similarly show the equity and fixed income portfolios, respectively.

Charts 3-5 are based on 51 monthly observations from July 1998 to September 2002. With a 90 per cent confidence interval, we can expect 5 per cent of the monthly excess return rates to lie below the lower bound, while 5 per cent lie over the upper bound.

**Chart 3: Actual and predicted risk for the Petroleum Fund. Percentage points**

![Chart 3: Actual and predicted risk for the Petroleum Fund. Percentage points](image)

**Chart 4: Actual and predicted risk for the Petroleum Fund’s equity portfolio. Percentage points**

![Chart 4: Actual and predicted risk for the Petroleum Fund’s equity portfolio. Percentage points](image)

For the total portfolio and the equity portfolio, too many observations lie outside the bounds of the confidence interval. This is an indication that the risk system has measured too low a risk. The opposite is the case for the fixed income portfolio. Here there are few excess return rates that fall outside the bounds of the confidence interval, which indicates that the risk system measures too high a risk for the fixed income portfolio.
Norges Bank’s management of market risk

Risk measurement is not an exact science. No single method or model can provide a good picture of the short- and long-term risk associated with investing in securities markets. Norges Bank therefore uses a broad-based set of methods to maintain tight control of market risk in the Petroleum Fund.

From the time when the investment strategy is established, emphasis is placed on putting together a portfolio consisting of many asset classes and securities. The aim is to keep risk to a minimum in relation to the expected return. When the investment strategy is implemented, a number of different approaches are then used to measure and control risk. These two methods for controlling risk complement one another.

The investment strategy

Norges Bank has defined a strategy for the type of management that offers the best possibility of achieving an excess return in relation to the benchmark index defined by the Ministry of Finance. See also the feature article “Strategy for achieving the best possible performance" published on the Petroleum Fund’s website. A key element in the strategy is to achieve the best possible tradeoff between expected excess return and expected risk. The most important strategic contribution to keeping risk low is the emphasis on taking many independent active positions rather than a few large positions. This approach is consistent with modern portfolio management theory, often called “The fundamental law of active management” (the concept stems from Grinold and Kahn, ‘Active Portfolio Management’, Irwin 1995).

The strategy is implemented in practice by delegating the individual management decisions to many decision-makers. At the end of 2002, for example, there were 24 external managers with a total of 40 external management mandates in the Petroleum Fund’s equity and fixed income portfolio. Decision-making is also delegated within each of these organisations - and all decision-makers pursue a general strategy of taking many independent positions. Moreover, the individual management mandates are selected on the basis of an overall analysis of risk properties; great emphasis is placed on selecting managers who complement one another.

Similar principles are applied to the organisation of internal management. There are many decision-makers and a number of different approaches - distributed among about 20 different fixed income and equity mandates. None of the internal management strategies represents a significant portion of the overall active management risk. Nor is there any coordination in the form of guidelines for a uniform market view that is to form the basis for internal management.

Chart 6, for example, shows first the distribution of active risk among the main types of fixed income management, with seven profit centres. One of the profit centres accounts for 47 per cent of the risk-taking. Within each profit centre, risk can be distributed further to individual mandates and positions. The figure to the right in the chart shows that the risk within the profit centre that accounts overall for most risk in fixed income management is distributed among a large number of individual positions, all of which are small.

Chart 6: Market risk in fixed income management in the Petroleum Fund, by profit centre and by individual positions within one of the profit centres.
Choice of method and systems for measuring risk
Since there is considerable uncertainty associated with risk measurement, Norges Bank cannot base its calculations on a single method of approach and model. A broad-based set of methods and models is used as a background for regular analyses of management risk.

Each year, the Executive Director of Norges Bank Investment Management allocates limits for active risk-taking to the heads of Fixed Income and Equities. The risk limits are defined in RiskManager, which is the model used to ensure that management takes place within the risk limits defined by the Ministry of Finance. The allocation of risk limits is based on investment plans from the heads of Fixed Income and Equities. The plans describe how it is intended that risk should be allocated to the various external and internal management activities. Both risk limits and risk-taking budgets are monitored constantly through the year.

In allocating risk limits, great emphasis is placed on testing the effect of different assumptions in the risk model. For example, the covariation between returns in different markets will typically be stronger in situations with an unusually weak return. For this reason, the consequences of stronger covariation than that estimated in the risk model are always investigated before risk limits are allocated.

Use of RiskManager alone could result in unpleasant surprises for Norges Bank in situations where the normal results with respect to low correlation between equity and fixed income management no longer held true. The consequence could be a sharp increase in the risk measured by the model, and hence a breach with the guidelines from the Ministry of Finance. By performing different types of stress testing and by introducing risk limits that are robust to several possible scenarios, we achieve a reduction of the risk connected to the choice of RiskManager as the primary model for measuring market risk.

Many other methods and models are used in both fixed income and equity management to compute and monitor risk. For example, the managers have other models than RiskManager for estimating the risk of the positions they take. Both equity and fixed income management thus compute alternative risk figures and monitor these daily. Risk is also limited by stringent rules regarding the size of each position and how long it can be maintained if it should result in a loss.

On balance, the use of models, the implementation of the investment strategy and the rules for monitoring individual positions provide very good safeguards so that market risk does not become unacceptably high or breach the risk limits set for the management of the Petroleum Fund.