NBIM
Norges Bank Investment Management

Tracking error as a measure of market risk

The Ministry of Finance has defined a clear mandate for Norges Bank's management of the Government Petroleum Fund. A benchmark portfolio of specified equities and bonds constitutes an important management and measurement tool. In its day-to-day management, Norges Bank may deviate slightly from the benchmark portfolio, but limits are placed on the magnitude of these deviations. The principal restriction on how much market risk Norges Bank can take in relation to the benchmark portfolio takes the form of an upper limit on expected tracking error. The Ministry of Finance has also imposed other types of restriction on relative market risk. In this article, the concept of market risk is discussed first. The concept of tracking error is then presented, along with a discussion of how strict the other restrictions on market risk are compared with the limit imposed on tracking error.

The benchmark portfolio is a hypothetical portfolio composed of a selection of securities from the various markets in which the Fund is permitted to invest. The composition of the benchmark portfolio reflects the strategic choices the Ministry of Finance has made for the management of the Fund with respect to distribution among currencies and markets, between equities and fixed-income instruments, and the distribution of securities within the various market segments. In its management, the Bank may deviate from the benchmark portfolio if this provides scope for achieving an excess return or more cost-effective management. The Ministry imposes two types of limits on the deviations that are permitted. First, intervals are specified for holdings of equities and fixed-income instruments, and for holdings in various markets. Second, a general limit is placed on risk relative to the benchmark portfolio. In practice it is this general restriction that effectively limits the market risk that may be taken.

An example of the restrictions on holdings in asset classes and markets is that Norges Bank may not invest more than 50 per cent or less than 30 per cent of the Fund's capital in equities. Similarly, limits are imposed on the deviations permitted in the regional distribution of the actual portfolio compared with the benchmark portfolio. Limits are also placed on the extent to which the average duration of the fixed income instruments the Fund actually holds may deviate from what the benchmark portfolio defines as neutral, and how large a holding the Fund may have in an individual company.

The disadvantage of such direct limits on allocation is that they do not take account of the covariation between the returns for the individual assets classes, markets and currencies. As a result, investments that in aggregate do not cause significantly greater fluctuations in the value of the Fund may be in conflict with the individual restrictions, while a number of minor deviations, each of them permissible individually, may lead to major fluctuations in the value of the Fund.

One approach to this problem is to impose restrictions in the form of limits on the fluctuations in the difference between the returns on the actual and benchmark portfolios that may be accepted as normal. The Ministry of Finance has chosen to impose such limits on Norges Bank, by requiring that the difference in returns that may normally be expected should not be too large. The Ministry is primarily interested in ensuring that Norges Bank's return is not significantly lower than that which can be achieved by shadowing the benchmark portfolio, but it is not possible to establish in advance a limit on how much lower Norges Bank's return will actually be. The Ministry of Finance's limit therefore concerns an expected deviation, in the form of an upper limit of 1.5 percentage points on expected tracking error (the standard deviation between the actual return and the benchmark return – see below). The Ministry of Finance has furthermore stipulated how this expected value is to be measured, on the basis of a statistical model of the risk associated with future market movements.

What is market risk?

The Bank for International Settlements (BIS) splits the concept of market risk into two components: general market risk and specific risk. General market risk pertains to changes in the market value of assets and liabilities as a result of broad movements in financial markets, such as changes in interest rates, equity prices, exchange rates and commodity prices. Specific risk pertains to changes in the market value of assets and liabilities due to factors other than those mentioned above. Examples are changes in borrowers' or issuers' credit rating, a change in the liquidity of a class of securities, or unusual local events, such as natural disasters or political events.

The Ministry of Finance controls the total risk assumed in the management of the Fund through its choice of benchmark portfolio, and by limiting the scope for deviation from this portfolio. Norges Bank's role as operational manager of the Fund is to implement the strategy in a prudent, cost-effective manner, which includes controlling the variations in the Fund's total value and the difference between the return the Bank actually achieves and the return on the benchmark portfolio.

Specific market risk is more difficult to model statistically. The discussion in this article is therefore limited to the general market risk associated with the Petroleum Fund in the form of fluctuations in the total value of the Fund and the difference between the return on the actual Fund and that on the benchmark fund. In the remainder of this article, the term "market risk" is thus used in this narrow sense.

Time horizons for risk measurement

Fluctuations in market values and return will be wider over a long time horizon than over a short one. It is therefore not possible to discuss the magnitude of the fluctuations that should be permitted without specifying the time period in question.

In the Revised National Budget for 1997, the Ministry of Finance writes: “It is natural to make a long-term investment horizon the basis for the Petroleum Fund, and to place emphasis on the objective of preserving the Fund’s international purchasing power.” The time horizon that should be used for evaluating the Fund’s investments depends on how long it will be until the need to draw on the Fund arises. Projections about future cash flows are available. They are naturally shrouded in uncertainty, but current assumptions indicate that it will not be relevant to draw on the Fund for more than a decade.

Brokers and investment banks are evaluated over a relatively short time horizon, measured in days or, at the most, weeks. These investors tend to use risk evaluation models based on daily data. Investors such as life insurance companies and securities funds are evaluated in a longer-term perspective, usually a year or more. The models that these longer-term investors use for risk evaluation are therefore often based on monthly data. In the remainder of this article, a time perspective of one year is assumed for risk measurement. Such a time horizon can be regarded as a compromise between the Fund’s long-term objectives and the need to maintain control of the ongoing changes in the Fund’s market value.

Risk measurement

The Government Petroleum Fund is a means of achieving policy objectives in various areas, not an end in itself. In a broad perspective, risk therefore relates to whether it is possible to attain these policy objectives. A slightly narrower approach is adopted here. An attempt is made to quantify variations in the total value of the Fund, and the difference that can be expected between the return on the Fund and that on the benchmark portfolio.

The expected value of the annual return provides some indication of the range in which the return will normally lie. The standard deviation is a measure of the breadth of this probable range around the expected value. As a measure of risk, it describes the spread in what can be called normal deviations from the expected return. The standard deviation of the return over a period of one year is frequently called the volatility of the return. Thus the volatility does not provide a direct indication of the likelihood of a very low return or heavy losses.

Another type of risk measure focuses on particularly unfavourable investment performance. For example, one could try to measure the level of a return that is so low that the actual result is only lower one time in a hundred. This is normally called the first percentile, and is an example of a quantile risk measure. Note that whereas volatility primarily provides information about the spread of normal results, quantiles provide information about particularly unfavourable results. These two risk measures complement one another.

The return on a portfolio can be measured either in currency, or as a percentage of invested capital. One risk measure that has been used increasingly over the past five years is “value at risk”. This is often defined as the first percentile or fifth percentile of the return measured in currency. Investment banks and brokers in particular use this measure, because it can easily be linked to a firm’s equity or risk capital requirements.

The return achieved by portfolio managers is often compared with the return on representative market indices or the results of other managers. The return is most commonly measured as a percentage of invested capital. It is common practice to measure the market risk the managers have taken by looking at the range within which the results would normally occur, in other words using the concept of volatility. One may look at the volatility of the absolute return on the portfolio, or the volatility of the difference in return from that of a market index or specific benchmark portfolio (relative return). The spread of normal differences between two return figures is called tracking error.

Different agents thus place emphasis on different dimensions of the probability distribution. Nevertheless, there is no reason to rely on one of these measures to the exclusion of all others. Both types of spread measure provide interesting information about the sort of result that can be expected or feared.

Characteristics of tracking error

In this section, a closer look is taken at the characteristics of tracking error as a measure of risk, and illustrations are provided in the form of numerical examples relating to the management of the Petroleum Fund.

The Ministry of Finance uses the concept to define the scope for manoeuvre in management. Norges Bank uses tracking error as an instrument in its day-to-day management. Tracking error is a precise concept, but it can be calculated and used in a number of ways. Ex ante estimates of tracking error are calculated on the basis of the composition of the actual and benchmark portfolios, using a statistical model of changes in interest rates and equity prices. The size of the estimated tracking error depends on two things: the actual differences between the two portfolios, and the statistical relationship between the returns on the assets in the actual and benchmark portfolios.

The tracking error will be zero if there is no deviation from the benchmark portfolio, and it will increase as deviations from the benchmark increase. The estimated volatility of the return on each individual security and the covariation between the returns on the various securities in the actual and benchmark portfolios are calculated by means of a statistical model. A positive covariation between two securities means that as a rule they will yield high returns at the same time, and low returns at the same time. Negative covariation means that the returns on the two securities will tend to move in opposite directions. Estimates of volatility and covariation are calculated with the aid of historical data. If it is assumed that the historical patterns will continue to apply in the future, tracking error can be used to indicate whether and to what extent the future return on the actual portfolio can be expected to deviate from the return on the benchmark portfolio.

The numerical examples in this section are based on covariation figures for sub-components of the Petroleum Fund’s benchmark portfolio, calculated from monthly return figures for sub-indices over the past ten years. Recent data have been given greater weights when covariation figures are calculated.

Relationship between absolute volatility and tracking error

When the Ministry of Finance selects a benchmark portfolio and imposes limits on tracking error, the volatility of the total

return on the actual portfolio is also restricted. The latter can be called absolute volatility. There is a close, but highly complex relationship between absolute volatility and tracking error. Suffice it to say that the absolute volatility of the actual return cannot be larger than the sum of the volatility of the return on the benchmark portfolio and the limit on tracking error.

Effect of the choice of basis currency on absolute volatility and tracking error

The measured variation in return will depend on the currency used as the basis currency. During the exchange market turbulence in 1998, for example, the estimated variation in the Fund's total return, measured in NOK, increased sharply because the exchange rate of the Norwegian krone against other currencies fluctuated widely. The objective of the management of the Petroleum Fund is to maintain the international purchasing power of the Fund. For this reason, a currency basket corresponding to the composition of the benchmark portfolio is normally used as the basis currency in calculations, eliminating the effect of fluctuations in the value of the krone.

Deviations in the composition of assets

The examples below have been constructed to illustrate the characteristics of tracking error. The deviations from the benchmark portfolio presented in the examples are therefore highly exaggerated by comparison with the deviations Norges Bank actually makes from the benchmark as part of its day-to-day investment management.

Table 1: Absolute and relative risk for a 55/45 distribution of fixed income assets and equities (percentage points)

Volume	NOK	Currency basket
Volatility, actual return	8.51	6.77
Volatility, benchmark return	8.04	6.15
Tracking error	0.70	0.70

Let us first assume that there has been a deviation along only one axis: instead of being 60/40, the relative distribution of fixed income instruments and equities in the benchmark portfolio is 55/45. Table 1 shows the magnitude of the absolute volatility and tracking error we would expect from such a deviation, in the light of historical market developments. A substantial portion of the absolute volatility for total return measured in NOK is currency risk. If we use the Fund's currency basket rather than NOK as the basis currency, the volatility of the total return will be reduced by almost one fourth. This is because the broad-based currency basket is less prone to wide fluctuations than a single currency. The tracking error, on the other hand, is unchanged, and as a general rule the tracking error level will be unaffected by the choice of basis currency.

How much of the variation in total return is due to the Bank's management?

The division of responsibilities in the management of the Petroleum Fund is clear: The Ministry of Finance specifies a benchmark portfolio and sets limits for Norges Bank's deviations from it. As delegating authority, the Ministry of Finance is naturally concerned with the level of and the fluctuations in the overall result: the sum of benchmark return and active return. To what extent do Norge Bank's dispositions influence the variation in the Fund's overall return?

In Table 1, the absolute volatility (measured in terms of the Fund's currency basket) is 6.77 percentage points, compared with 6.15 percentage points for the benchmark portfolio. The deviation thus caused a ten per cent increase in the volatility of the overall return. In this case, the covariation of the active return with the benchmark portfolio return is fairly strong. A more normal situation is that when making deviations, the manager remains relatively neutral to developments in the benchmark portfolio. In such case, the active return will be almost independent of the return on the benchmark portfolio, and the absolute volatility of the actual portfolio will not be much higher than that of the benchmark portfolio. If there is full independence, and the Bank chooses a deviation that results in a tracking error of 1.5 percentage points in addition to the benchmark portfolio's absolute volatility of 6.15 percentage points, the absolute volatility of the overall return will be 6.33 percentage points. In this example, some 97 per cent of the total variation in the Fund's value is thus associated with the Ministry of Finance's choice of benchmark portfolio.

This illustrates the fact that the variation in the overall return on the Petroleum Fund is largely determined by the Ministry of Finance through its specification of the benchmark portfolio and the allowed limits for deviation from this portfolio.

Direct limits for distribution of assets and tracking error

In addition to the limit on tracking error, the Ministry's Regulation on the Management of the Petroleum Fund stipulates other limits. As operational manager, Norges Bank must observe these limits (see Table 2). In the following, a closer look will be taken at how the direct restrictions on the distribution of assets and the general limit on tracking error are related.

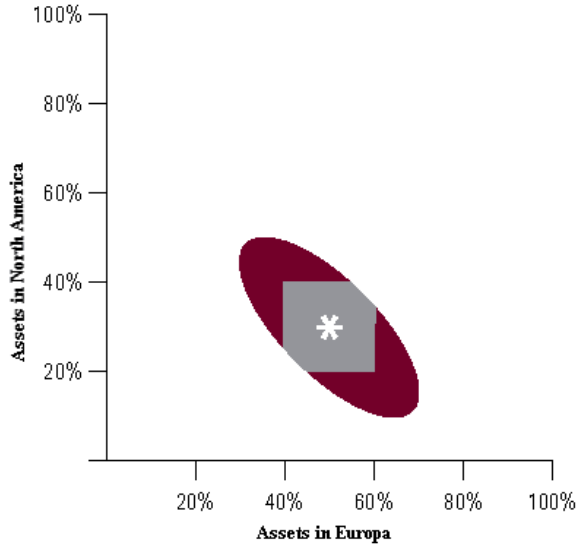
Table 2: Direct restrictions on management in the Petroleum Fund Regulation

Asset distribution	Equities: 30 to 50 per cent, Fixed income instruments: 50 to 70 per cent
Regional distribution	Asia: 10 to 30 per cent, Europe: 40 to 60 per cent, North America: 20 to 40 per cent
Modified duration (interest rate sensitivity)	3 to 7
Maximum holding in company	Not more than 1 per cent of equity capital

Let us look first at the restrictions on distribution between asset classes. Taking the Fund's benchmark portfolio as the starting point, it is assumed that the relative weights of equities and fixed income instruments are altered, all else remaining unchanged. Tracking error is calculated relative to the Fund's benchmark portfolio for each deviation. It can be demonstrated that the tracking error in this example will be proportional to the absolute deviation from the benchmark portfolio's 60/40 distribution between fixed income instruments and equities. A composition of 55/45 will thus result in the

same tracking error as a 65/35 composition, and a 70/30 composition will give rise to a tracking error twice as high as a 65/35 composition. Whereas the direct limits are binding, in that a maximum absolute deviation in asset class weights of 10 percentage points is allowed, the general risk cap of 1.5 percentage points tracking error will only be breached at an absolute asset class weight deviation of 10.7 percentage points. But this is contingent on there being no other deviations from the benchmark portfolio. In practice, it is the general risk cap that will be binding.

Chart 1: Regional distribution



Analysing deviations from the regional distribution is more complicated, because there are two variables here: the combined weight for the three regions must total 100 per cent, so it is sufficient to vary the weights of two of the regions.

Chart 1 shows how the relative portions invested in Europe and North America may vary within the limits imposed by the guidelines. The red ellipse shows all regional distributions for which the tracking error is 1.5 percentage points or less. The blue hexagon marks all regional distributions that satisfy the requirements implicit in the direct limits on portions in each region in Table 2. The regional weights in the benchmark portfolio are marked with a white star. The blue hexagon lies within the ellipse, which means that the direct limits form the tightest restriction. But it is assumed once again that the actual portfolio does not deviate in any other way from the benchmark portfolio. If it does, the result will be much less clear.

It may be concluded that the direct limits appear to be more restrictive than the general risk limit in these simple cases with only one type of deviation from the benchmark portfolio. But the examples of deviations are naturally not representative of the manner in which management is conducted from day to day. There will always be a large number of deviations from the benchmark portfolio contributing to the general market risk. And as a rule it will be the limit on expected tracking error that actually restricts the scope for manoeuvre in management. It is difficult to conceive of the direct limits on market shares or similar restrictions functioning as effective constraints in practice.

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